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A method is presented for calculating the combustion of a polydisperse fuel inside a fluidized bed and above the bed.

The distribution of coal particles by size after crushing can usually be described by the Rosen-Rammler equation

$$R = \exp[-(x/x_0)^n]. \quad (1)$$

To simplify the analysis, we can assume (at least when the fuel is charged above the level of the bed) that coal particles with a diameter  $x$  less than the critical diameter  $x_1$ , determined by the free-fall velocity are entrained by the flow of flue gases and burn in the space above the bed. Particles of the size  $x > x_1$  enter the volume of the bed.

Considering that, under actual conditions, the maximum particle size in the initial screened batch is always limited to a certain value  $x_2$  (screen size, gap between the crusher rolls, etc.), the distribution function of particles supplied for combustion in the volume of the fluidized bed can be represented in the form

$$R_0 = \frac{\exp[-(x/x_0)^n] - \exp[-(x_2/x_0)^n]}{\exp[-(x_1/x_0)^n] - \exp[-(x_2/x_0)^n]}, \quad (2)$$

or

$$y_0 = -\frac{dR_0}{dx} = \frac{nx^{n-1} \exp[-(x/x_0)^n]}{x_0^n \{\exp[-(x_1/x_0)^n] - \exp[-(x_2/x_0)^n]\}}. \quad (2')$$

The initial distribution function of particles supplied for final combustion in the space above the bed is accordingly determined as

$$R_0^* = \frac{\exp[-(x/x_0)^n] - \exp[-(x_1/x_0)^n]}{1 - \exp[-(x_1/x_0)^n]}, \quad (3)$$

or

$$y_0^* = -\frac{dR_0^*}{dx} = \frac{nx^{n-1} \exp[-(x/x_0)^n]}{x_0^n \{1 - \exp[-(x_1/x_0)^n]\}}. \quad (3')$$

**1. Combustion in the Fluidized Bed.** During steady-state combustion in the volume of the bed, a certain coal particle concentration is established. Accordingly, a particle-size distribution function differing from  $y_0$  and corresponding to this concentration is also established. It can be assumed that, with intensive mixing, particles of all sizes are distributed uniformly over the height of the bed.

As the effective oxygen concentration in the bed we take the mean value over the bed height, calculated from the expression

$$\bar{C} = C_0(1 - 0.5 \alpha_{be}), \quad (4)$$

where  $C_0$  is the oxygen concentration in the incoming gas stream (under the gas-distributing grate). The excess air coefficient  $\alpha_{be}$ , calculated for the fuel entering the bed, differs from the excess air coefficient  $\alpha$  calculated for all of the fuel which enters the combustion chamber and is connected with the latter by the relation

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$$\alpha_{be} = \frac{\alpha}{\exp[-(x_1/x_0)^n]} \quad (5)$$

During combustion, the change in the mass  $M_x$  of coal particles of a given size  $x$  is equal to the difference between the rate of delivery of particles to the bed and the rate of their combustion in the bed, with a volume  $V$ . The combustion rate (the amount of carbon  $B_x^{cb}$  of the given fraction  $x$  which disappears from the volume of the bed  $V$  per unit of time) is proportional to the specific surface of the particles of this fraction  $S_1$  and the rate of carbon consumption  $K_s$  per unit of surface:

$$B_x^{cb} = K_s V S_1 dx = K_s \frac{6V\rho_i(1-\varepsilon)z}{\rho x} dx \quad (6)$$

In low-temperature combustion, the role of the reaction of carbon dioxide reduction is unimportant. Thus, we will assume that only the reaction  $C + O_2 = CO_2$  takes place. Under fluidized-bed conditions, the rate of carbon consumption in the general case depends on both the kinetics of the chemical reaction and diffusive mass transport through the boundary layer to the burning particles, as well as on mass transfer between the high- and low-density phases of the bed. In the practically interesting case of fluidization of coarse particles, the bubble velocity is lower than the velocity of gas filtration through the continuous phase, i.e., the "bubble-flow" regime prevails. Then mass transport of oxygen between the high- and low-density phases of the bed ceases to be a determining factor, and the specific rate of oxygen consumption can be calculated from the expression

$$K_s = \frac{12}{32} \bar{C} \frac{1}{\frac{1}{k} + \frac{x}{Sh D}} \quad (7)$$

The amount of carbon of the given fraction  $x$  entering the bed, expressed through the excess air coefficient  $\alpha$  and the initial particle-size distribution  $y_0$ , is determined from the formula

$$B_x = \frac{wF}{\alpha v_0} y_0 dx \quad (8)$$

The change in the mass  $dM_x$  of a given fraction over the period  $\tau$  during combustion in the bed is determined by the change in the total mass concentration of carbon in the bed  $z$  and the particle-size distribution function:

$$\frac{dM_x}{d\tau} = V\rho_i(1-\varepsilon) \frac{d(zy)}{d\tau} dx = V\rho_i(1-\varepsilon) \left( y \frac{\partial z}{\partial \tau} + z \frac{\partial y}{\partial \tau} + z \frac{\partial y}{\partial x} \frac{\partial x}{\partial \tau} \right) dx \quad (9)$$

where the change in particle size over time is connected with the specific rate of carbon consumption by the relation

$$\frac{\partial x}{\partial \tau} = - \frac{2K_s}{\rho} \quad (10)$$

For the steady-state combustion of a polydisperse fuel,  $\partial y/\partial \tau = \partial z/\partial \tau = 0$ . Equating the change in the mass of a given fraction  $x$  (9) to the difference between the amount of carbon of the same fraction which has entered the bed (8) and which has undergone combustion (6), we obtain the equation for the change in the particle-size distribution function in the bed  $y$ :

$$\frac{dy}{dx} = \frac{3}{x} y - \frac{\varphi(x)}{z} \quad (11)$$

where

$$\varphi(x) = \frac{w\rho y_0}{2K_s H \rho_i \alpha_{be} v_0 (1-\varepsilon)}$$

The solution of Eq. (11) has the form

$$y = x^3 \left( C_1 - \int \frac{\varphi(x)}{z} x^{-3} dx \right) \quad (12)$$

where it is necessary to use the following normalization equation to determine the constant of integration  $C_1$

$$\int_{x_1}^{x_2} y dx = 1. \quad (13)$$

Here, we integrate from the minimum possible particle size with regard to entrainment conditions  $x_1$  to the maximum size in the initial fuel  $x_2$ .

To determine the mass concentration  $z$  of fuel in the bed in Eq. (12), we need the balance equation for the arriving and burned quantities of fuel

$$\frac{w}{\alpha_{be} U_0} = H \int_{x_1}^{x_2} K_s S_i dx = H \frac{6z\rho_i(1-\varepsilon)}{\rho} \int_{x_1}^{x_2} \frac{K_s y}{x} dx. \quad (14)$$

Equation (14) was integrated with allowance for conditions (12) and (13) on a computer for different solid-fuel sizes. The calculations showed that the content of coarse particles in the fluidized bed is high relative to the content of such particles in the initial fuel (Fig. 1). Meanwhile, this effect is manifest to a greater extent, the larger the maximum particle size  $x_2$  in the initial fuel. Thus, given the same crushing index  $x_0$  for the initial fuel ( $x_0 = 3.25$  mm) with  $x_2 = 7$  mm, mean particle size  $\bar{x}$  in the bed turned out to be 4.4 mm. At  $x_2 = 13$  mm,  $\bar{x} = 7$  mm. This is to be expected, since fine particles burn more rapidly than coarse particles.

The steady-state value of fuel concentration in the bed  $z$  (Fig. 2) decreases with an increase in the excess air coefficient  $\alpha$ . Here, the greater  $x_2$  and, accordingly, the coarser the particles in the bed, the greater must be the fuel concentration in the bed in order to achieve the same degree of combustion.

Comparison of the calculated results with experimental data we obtained in the combustion of Irsha-Borodin coal showed that the theoretical concentrations are somewhat higher than the experimental values for realistic values of the combustion rate constant. This suggests that the granulometric composition of Irsha-Borodin coal burned in a fluidized bed is significantly influenced by the crushing of the particles — especially the coarse particles. This is seen in the combustion of narrow fractions of this coal in a fluidized bed.

2. Combustion in the Space Above the Bed. The total flow of particles delivered to the above-bed space for final combustion consists of the monodisperse flow which has undergone final combustion in the bed to the size  $x_1$  and then been transferred to the above-bed space and the polydisperse flow of particles of size  $x < x_1$  which directly entered this space during charging of the fuel. Here, the initial particle-size distribution of the fuel is  $y_0^*$ . If we take this approach, then we should calculate the mechanical underfiring for these two components (monodisperse flow and polydisperse flow).

The combustion of polydisperse coal particles in the above-bed space can be calculated by the method developed in [1, 2], where an examination of the mass balance led to an equation for the change in the particle-size distribution function in a pulverized-coal flow

$$\frac{\partial y}{\partial x} - \frac{\partial y}{\partial \tau} \frac{\rho}{2K_s} = \frac{3y}{x}. \quad (15)$$

In contrast to the studies [1, 2], which examined the combustion of finely dispersed coal dust moving at the velocity of the flow, we need to consider the relative velocity of the particles in the case of the combustion of fairly coarse particles in the above-bed space. Since the free-fall velocity depends on particle size, the true velocity of the particles, calculated as the difference between the velocity of the gas and the free-fall velocity  $u = w - w_y$ , will also be a function of the size of the coal particles. The free-fall (entrainment) velocity can be determined from the Todes formula

$$Re_y = \frac{Ar}{18 + 0.6 \sqrt{Ar}}. \quad (16)$$

Calculations showed that for the particles accounting for most of the mechanical underfiring in the combustion of coal in fluidized-bed furnaces ( $x = 0.3-1$  mm), the dependence of free-fall velocity on particle diameter can be assumed to be linear. Then particle velocity is determined as a function of particle diameter by the expression

$$u = w - k_1 x. \quad (17)$$

Here, the dimensional coefficient  $k_1$  is found by approximating Eq. (16) in the range of particle size  $x$  which is of interest. The time of particle combustion on an elementary segment of length is connected with the coordinate  $h$  by the relation

$$d\tau = dh/(w - k_1 x). \quad (18)$$

Then the equation for the particle-size distribution function in the combustion of polydisperse fuel has taken the form

$$\frac{\partial y}{\partial x} - \frac{\rho}{2K_s} (w - k_1 x) \frac{\partial y^*}{\partial h} = \frac{3y^*}{x}. \quad (19)$$

Following [1], we obtain the granulometric characteristic of the fuel burned in the flow in relation to the parameter  $t$ :

$$y^* = \frac{nx^3 t^{n-4}}{x_0^n} \frac{\exp[-(t/x_0)^n] - \exp[-(x_1/x_0)^n]}{1 - \exp[-(x_1/x_0)^n]}. \quad (20)$$

The parameter  $t$  is the root of the expression

$$x_0^3 - t^3 + \left( \frac{3DSh}{2k} - \frac{3w}{2k_1} \right) (x_0^2 - t^2) - \frac{3wDSh}{k_1 k} (x_0 - t) + \frac{9DShz_1}{4\rho k_1} = 0, \quad (21)$$

obtained from the solution of the characteristic equations of Eq. (19) for the particle-size distribution function.

The fraction of polydisperse entrained particles (of the total mass of entrained particles) which has not been completely burned at the distance  $h$  from the level of the fluidized bed can be determined as

$$q_4^* = \int_0^{\bar{x}_1} y^* dx. \quad (22)$$

Since the value of the parameter  $t$  depends on the concentration of oxygen, which changes over the height of the combustion chamber and is related to the total amount of mechanical underfiring, then system (20)-(22) can be solved only on a computer. In integrating Eq. (22), as  $\bar{x}_1$  we take the maximum particle size at the given height above the bed. It is determined from calculation of the combustion of the monodisperse flow of entrained fuel.

With steady combustion and in the absence of particle fragmentation in the bed, it can be assumed that the number of entrained particles burned to the final size  $x_1$  is equal to the total number of particles charged into the bed. Since the number of particles of the given size per kg of fuel entering the bed is

$$N(x) = \frac{6y_0}{\pi\rho x^3}, \quad (23)$$

then the total number of particles charged per  $m^2$  of bed surface can be represented as follows:

$$N = \frac{w}{\alpha v_0} \int_{x_1}^{x_2} \frac{6y_0}{\pi\rho x^3} dx. \quad (24)$$

Then the mass flow of entrained monodisperse particles from  $1 m^2$  of bed surface is determined as

$$G_1 = N \frac{\pi\rho}{6} x_1^3, \quad (25)$$

and the flow of entrained polydisperse particles is determined as

$$G_2 = \frac{w}{\alpha v_0} [1 - \exp(-x_1/x_0)]. \quad (26)$$

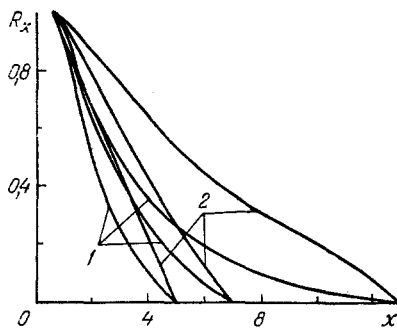


Fig. 1

Fig. 1. Theoretical integral granulometric characteristics of the fuel,  $T_{fb} = 1123^\circ\text{K}$ ,  $w = 1.6$  m/sec: 1) initial fuel entering the bed; 2) fuel inside the bed during combustion.  $x$ , mm.

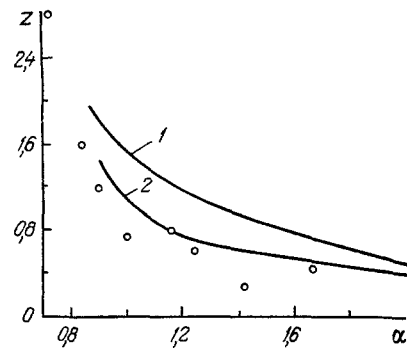


Fig. 2

Fig. 2. Dependence of the relative mass concentration of fuel  $z$  in the bed on the excess air coefficient  $\alpha$ ,  $T_{fb} = 1123^\circ\text{K}$ ,  $w = 1.6$  m/sec; points represent experimental results (the fuel was Irsha-Borodin brown coal,  $x_2 = 7$  mm,  $x_0 = 3.25$  mm); curves show calculated results; 1)  $x_2 = 7$  mm,  $x_0 = 3.25$  mm; 2) 5 and 2 mm.

In this case, mechanical underfiring can be calculated as the sum of the underfiring of the two flows referred to the total initial mass flow:

$$q_4 = (q_4''G_2 + q_4'G_1) / (G_2 + G_1). \quad (27)$$

Mechanical underfiring in combustion in the above-bed space  $q_4' = (\bar{x}_1/x_1)^3$  is determined rather simply from the solution of the equation

$$(w - k_1x) \frac{d\bar{x}_1}{dh} = - \frac{2K_s}{\rho} \quad (28)$$

with the initial condition  $h = 0$ ,  $\bar{x} = x_1$ .

System (20), (22), (27) was solved on a computer. Here, it was considered that the oxygen concentration is connected with the running value of mechanical underfiring by the relation

$$C = C_{ox} \left( 1 + \frac{q_4 - 1}{\alpha_{ab}} \right), \quad (29)$$

where  $C_{ox} = C_0(1 - 1/\alpha_{be})$ . The excess air coefficient  $\alpha_{ab}$ , calculated for the fuel entering the above-bed space, is determined from the expression  $\alpha_{ab} = (1/\alpha - 1/\alpha_{be})^{-1}$ .

Calculations gave us the particle-size distribution function in the above-bed space (Fig. 3), integral granulometric characteristics, and the change in mechanical underfiring over the height of the combustion chamber. Comparison of the theoretical change in mechanical underfiring over the chamber height in relation to the excess air coefficient  $\alpha$  with the results of experiments conducted on a laboratory unit showed satisfactory agreement (Fig. 4).

The above-described method of calculating the combustion of solid fuel in a fluidized bed makes it possible to evaluate the fuel concentration in the bed with different regimes for factors in the process. Given the fuel consumption, ash content, and concentration of fuel in a bed, it is possible to calculate heat loss with mechanical underfiring during discharge of bed particles (to maintain the prescribed bed height).

The calculations showed that the initial granulometric characteristics of the fuel have a significant effect on the fuel concentration in the bed. For example, a reduction in the maximum size of the coal particles from 7 to 5 mm led to a reduction in fuel concentration by a factor of 1.5, other conditions being equal. The method of calculation described here also makes it possible to evaluate heat losses with mechanical underfiring of finely dispersed coal dust entrained from the bed and to select the optimum height of combustion chamber for reducing underfiring to an acceptable level.

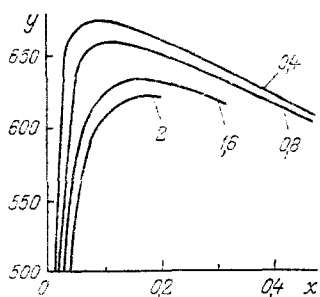


Fig. 3

Fig. 3. Theoretical distribution function  $y$ ,  $1/m$ , of particle size in the above-bed space,  $\alpha = 1.05$ ,  $x_2 = 7$  mm; the numbers next to the curves denote the height over the level of the fluidized bed, m.

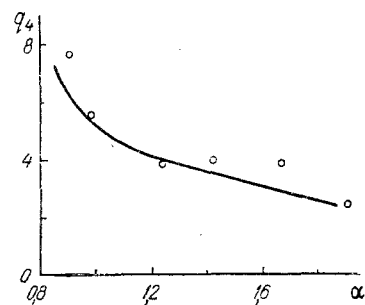


Fig. 4

Fig. 4. Dependence of mechanical underfiring  $q_4$ , %, on the excess air coefficient  $\alpha$ ,  $T_{fb} = 1123^\circ K$ ,  $w = 1.6$  m/sec,  $x_2 = 7$  mm, height over the level of the fluidized bed 0.8 m, fuel - Irsha-Borodin brown coal; the points denote experimental results, while the curves show calculated results.

#### NOTATION

$C$ , effective oxygen concentration in the bed;  $C_{Ox}$ , oxygen concentration at the bed outlet;  $D$ , coefficient of molecular diffusion;  $F$ , surface of fluidized bed;  $g$ , acceleration due to gravity;  $H$ , height of fluidized bed;  $k$ , rate constant of chemical reaction;  $K_g$ , specific rate of oxygen consumption;  $n$ , polydispersity index in the Rosen-Rammler equation;  $N(x)$ , number of particles of the size  $x$  per kg of fuel;  $N$ , total number of fuel particles charged per  $m^2$  of bed surface;  $q_4$ , mechanical underfiring;  $R_0$ ,  $R_0^*$ , integral granulometric characteristics of the initial fuel entering the bed and the above-bed space, respectively;  $V_0$ , theoretically necessary quantity of air for combustion of 1 kg of carbon;  $S_i$ , specific surface of coal particles of a prescribed fraction;  $V$ , volume of bed;  $w$ , velocity of gas flow calculated for the free cross section of the furnace;  $w_v$ , free-fall velocity of the particles;  $x$ , running size of coal particle;  $x_0$ , mean mass size of coal particles in the initial fuel;  $x_2$ , maximum size of coal particles in the initial fuel;  $x_1$ , maximum size of coal particles entrained from the bed;  $y_0$ ,  $y_0^*$ , particle-size distribution function in the initial fuel entering the bed and the above-bed space, respectively;  $z$ , relative mass concentration of fuel in the bed;  $\beta$ , mass-transfer coefficient;  $\epsilon$ , porosity of fluidized bed;  $\rho$ ,  $\rho_i$ ,  $\rho_f$ , density of coal particles, particles of inert material, and flue gases, respectively;  $\gamma$ , kinematic viscosity of the flue gases;  $Re = wx/v$ , Reynolds number;  $Sh = \beta x/D$ , Sherwood number;  $Ar = (gx^3/v^2)(\rho - \rho_f)/\rho_f$ , Archimedes number.

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